SDP relaxation of Maxcut

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The text files are formatted as follows: first row is two numbers representing the number of nodes and the ground truth maxcut, respectively. From then on, each row consists of three numbers, representing that there is an edge connecting node (first number) and node (second number), with edge weight (third number).

I adapted the following SDP relaxation from section 1.2.4 in

https://web.stanford.edu/class/ee364b/lectures/relaxations.pdf. a_{ij} is the edge weight connecting node i and node j, which is simply the third number as I described above, or zero if there are no connections between node i and node j.

We let $W \in \mathbf{S}^*$ be a matrix defined by $W_{ij} = -a_{ij}$ if $i \neq j$ and $W_{ii} = \sum_{j=1,\ldots,n} a_{ij}$ Note that the matrix W is here positive semidefinite. The problem is then formulated as:

maximize $\quad x^T W x$

$$\text{subject to } x_i^2 = 1, \quad i = 1, \ldots, n$$

with variable $x \in \mathbf{R}^n$. Thus, MAXCUT is a special case of the partitioning problem.